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# Formation of vortex rings from falling drops

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A study of the formation of vortex rings when a liquid drop falls into a stationary bath of the same liquid has been made. The investigation covered liquids with a wide range in surface tensions, densities and viscosities. The results confirm the reported existence of optimum dropping height from which the drop develops into a superior vortex ring. The optimum heights are analysed, by a photographic study, in terms of the liquid drop oscillation. It is found that vortex rings are formed best if the drop is spherical and changing from an oblate to a prolate spheroid at the moment of contact with the bath. A Reynolds number has been determined for vortex rings produced at optimum dropping heights; these numbers are approximately 1000. A possible mechanism for the ring formation is suggested.

## Introduction

Vortex rings were first reported by Rogers (1858) in a paper entitled 'On the formation of rotating rings of air and liquids under certain conditions of discharge'. A few years later Tait gave a set of lecture demonstrations on the formation of vortex rings using cylinder discharge. A number of interesting experiments using this method are described by Thomson (1867), Ball (1868) and Northrup (1912).

An alternative method of vortex ring production, namely from a liquid drop falling into a liquid bath, has attracted little attention. The pioneering work and only substantial treatment is that of Thomson & Newall (1885). They showed that the quality of the vortex ring, conveniently measured as the depth of penetration, changes considerably with changes in dropping height. Below the splashing limit, excellent vortex rings are formed from dropping heights which are equivalent to one drop oscillation apart. They gave no indication of the shape of drop which produced the best rings.

Thomson & Newall also proposed a mechanism for the vortex ring formation as follows. The drop penetrates the bath surface retaining its shape and, when just inside the bath, it experiences at its surface a finite alteration in tangential velocity over a very small distance. This gives rise to a vortex film. The drop tends to become disk-shaped, and simultaneously the vorticity diffuses inwards and outwards. If the viscosity is such that when the drop is in a disk shape it is full of vortex motion, then the unstable vortex disk will break up into a stable vortex ring.

Except for its role in determining the oscillation time of the drop, surface tension was assumed to play an insignificant part. Kutter (1916) has used the dependence of ring formation on drop oscillation to measure surface tension of liquids. He too did not know what drop shape produces the best rings.

Interest in vortex rings has recently revived with the discovery of macroscopic quantum effects, in the form of vortex lines and rings, in liquid helium. Edwards (1965) has reported the probable detection of quantized vortex rings produced in a helium II bubble chamber using the cylinder discharge technique. It seemed reasonable to try to produce vortex rings in helium by the drop method. The present results were obtained in classical liquids in order to understand the process more fully.

## Experimental

The apparatus basically consisted of three parts: a tip on which to form pendent liquid drops, a liquid bath at rest into which the drop falls and some detector to indicate the formation of vortex rings.

Various tips of brass and glass were used, flat, conical or hemispherical. These were attached to a lift mechanism allowing the height of the tip above the bath to be varied conveniently. A wick caused liquid to trickle on to the tip to form drops at a suitable slow rate. The liquid bath was contained in a clear glass graduated cylinder, large enough to minimize wall effects. Dyes of Sudan IV or methylene blue rendered the vortex rings visible. The surface tension of the dyed and undyed liquid—measured by Jaeger's method—differed by less than 4%. In some cases, particularly the cryogenic liquids, a thin layer of small particles was placed in a dish a few centimetres below the bath surface. If a vortex ring approached the dish with reasonable velocity, then the particles were displaced, leaving a ring imprint. A water vortex ring easily shifted iron filings on a dish 10 cm below the bath surface.

In a typical trial, the bath was filled and left standing until the liquid came to rest. With the tip at a fixed height above the bath, a pendent drop of dyed liquid was slowly formed and allowed to fall into the bath. If a vortex ring formed, the distance ($d$) which the ring travelled before stopping or breaking up was recorded. This procedure was repeated five times for each dropping height ($h$). Dropping heights ranged from zero to the splashing limit in roughly 2 mm intervals. For a given $h$ an average value of $d$ was found and a graph of $h$ versus $d$ plotted. Drop volumes were measured for the dyed fluid during each trial.

Close-up pictures were taken of a water drop during its first two oscillations after release from a tip. The drop was side illuminated by a xenon flash stroboscope at a frequency of 15 c/s and with an effective flash duration of about 1 ms. In addition, velocities and dimensions of the vortex rings were obtained from photographs of the rings, taken using fluorescent dyes and illuminating the rings from above. A polaroid camera with 3000 A.S.A. film was used in each case.

## Results and discussion

### *Optimum dropping height*

A liquid drop falling into a bath of the same liquid will usually form a vortex ring for all dropping heights ($h$) of a few centimetres. However, for certain dropping heights the vortex ring produced travels much farther in the bath than for other heights (see figure 1). A convenient measure of the quality of a vortex is the distance ($d$) which it travels. This distance may be as much as 20 cm for water.

Characteristic $d$–$h$ behaviour is shown in figure 2 for water and ethyl alcohol. Note that $h$ is measured from the centre of the drop as it detaches from the tip to its centre as it contacts the bath surface. Most liquids portray at least one local maximum in $d$ for values of $h$ in the range 1 to 3 cm. The dropping heights corresponding to the local maxima will be called optimum heights and denoted by

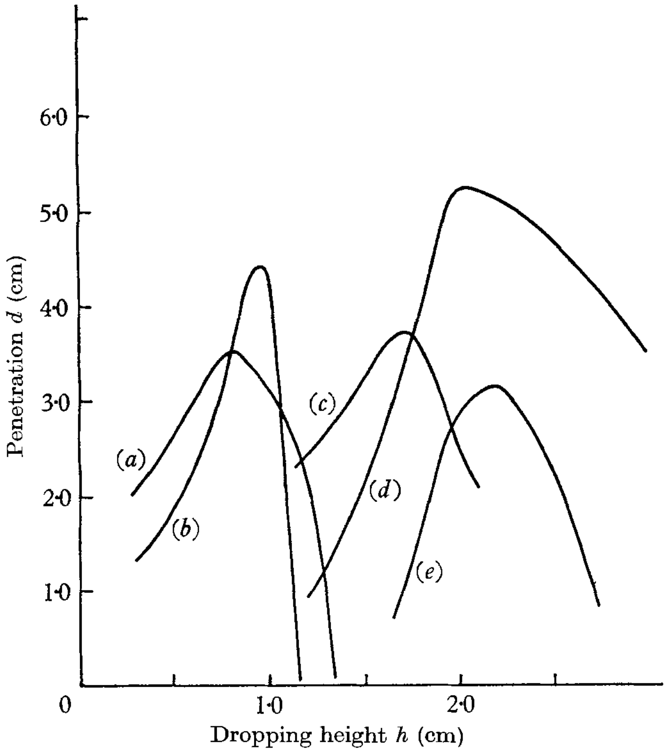


FIGURE 1. $d$–$h$ plot showing first optimum height for: (a) Freon 113, (b) Freon 114–B2, (c) methyl alcohol, (d) benzylamine, and (e) butyl alcohol, using hemispherical tip. Experimental points are omitted for clarity.

$H_1$, $H_2$, etc. The exact shape of the $h$–$d$ curve is not important; the position of the peaks is. Since the peaks are fairly broad, and an average of several trials at each $h$ is necessary to get meaningful results, values of $H_1$, $H_2$, etc. can only be determined to about $\pm 1$ mm.

A list of the various liquids used is given in table 1 together with some relevant properties. Data on the drops produced by these liquids, using a hemispherical glass tip, and the values of $H_1$ are also tabulated. The oscillation time ($\tau$) of a drop was calculated using the measured volume, known density $\rho$ and the equation

$$\tau = \left(\frac{3\pi}{8}\frac{\rho V}{T}\right)^{\frac{1}{2}},$$

where $T$ is the surface tension.

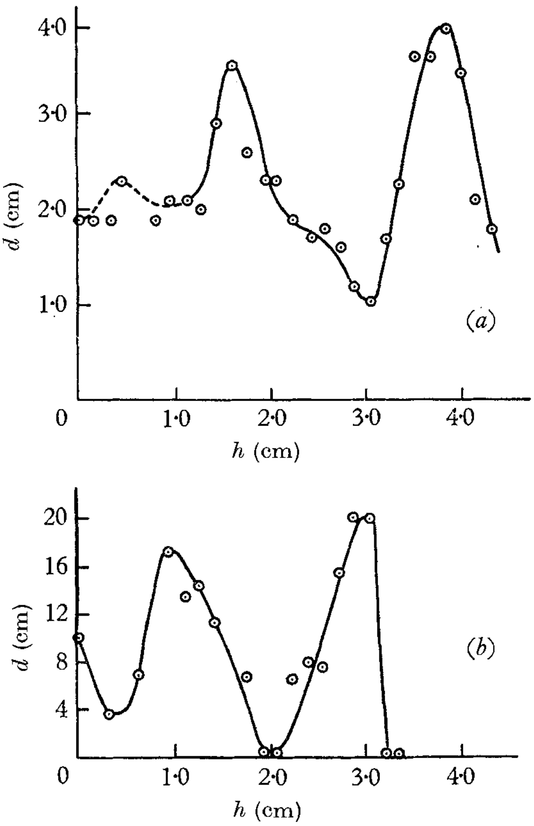


FIGURE 2. Characteristic $d$–$h$ plot for ($a$) ethyl alcohol and ($b$) water showing two optimum heights, using conical brass tip.

| Liquid | $T$ (dynes/cm) | $\rho$ (g/cm$^3$) | $T/\rho g \times 10^2$ (cm$^2$) | Volume $\times 10^2$ (cm$^3$) | $H_1$ (cm) |
|---|---|---|---|---|---|
| Water | 72·8 | 1·00 | 7·43 | 9·6 | 2·4 |
| Formamide | 58·2 | 1·13 | 5·25 | 6·9 | 2·0 |
| Benzylamine | 39·5 | 0·98 | 4·12 | 6·7 | 2·0 |
| Benzene | 28·9 | 0·88 | 3·35 | 4·5 | 2·0 |
| Acetone | 26·2 | 0·79 | 3·39 | 5·5 | 0 |
| Butyl alcohol | 24·6 | 1·04 | 2·41 | 4·5 | 2·2 |
| Ethyl acetate | 23·9 | 0·90 | 2·71 | 3·7 | 1·7 |
| Freon 113 | 23·0 | 1·57 | 1·50 | 1·4 | 0·8 |
| Methyl alcohol | 22·6 | 0·81 | 2·85 | 4·6 | 1·7 |
| Freon 11 | 22·0 | 1·48 | 1·49 | 1·8 | 0 |
| Ethyl alcohol | 21·4 | 0·79 | 2·67 | 4·5 | 1·9 |
| Freon 114–B2 | 21·0 | 2·16 | 0·99 | 0·98 | 1·0 |
| Isopentane | 13·8 | 0·62 | 2·26 | 4·1 | 0 |
| Nitrogen | 8·3 | 0·81 | 1·05 | 1·2 | 1·3 |
| Helium (1·8 °K) | 0·32 | 0·15 | 0·22 | 0·12 | — |
| Helium (4·2 °K) | 0·08 | 0·13 | 0·06 | — | — |

TABLE 1. Properties of liquids used, drop volumes and first optimum height $H_1$ (hemispherical tip)

Thomson & Newall in their original work noticed that the optimum heights are equivalent to one drop oscillation apart. We have confirmed this observation in the case of water, benzene, ethyl alcohol and liquid nitrogen. However, some liquids of low viscosity, e.g. Freon 11 (cooled to 0 °C) and acetone yield a double peak in the *d–h* plot rather than a single one at the first optimum. In order to understand this we undertook the vortex ring velocity measurements and the Reynolds number determinations.

Other anomalous effects have been observed. Benzene and methyl alcohol drops frequently produce two similar rings, travelling at a small angle to the vertical, for dropping heights smaller than the first optimum. Possibly the extreme oblateness of the drop is responsible. Also, we have found that liquids near their boiling points, e.g. Freon 11 and isopentane, only produce good rings from essentially zero dropping height. Evaporation effects are presumably strongly influencing the internal velocity field of the drop. The importance of this velocity field will be explained later. Freon 11 when cooled to 0 °C produces good rings at finite dropping heights.

The data of table 1 show a general trend; $H_1$ decreases as the value of $T/\rho g$ decreases. So far vortex rings have not been detected in liquid helium by the drop method.

### *Correlation between optimum heights and drop oscillation*

From the photographs of a falling water drop, the eccentricity may be obtained as a function of distance of fall. These are shown in figure 3*a*, where $a$ and $b$ are the horizontal and vertical diameters respectively. Thus $a/b > 1$ corresponds to an oblate spheroid, $a/b < 1$ to a prolate spheroid and $a/b = 1$ to a sphere. Figure 3*b* is the *d–h* characteristic of the resulting vortex ring. Remembering that $h$ is measured from centre to centre, it is clear that vortex rings are best formed when the drop shape is spherical or near spherical as it touches the bath surface. In addition, it is clear that, at the moment of contact, the drop must be changing from an oblate to a prolate spheroid.

The reason for this may be understood by considering the internal velocity field of the drop. This is shown in figure 3*c*, for the spherical configurations. For the drops which produce optimum vortex rings the internal velocity field opposes the impact, thus tending to preserve the spherical shape. On the other hand, if the drop were changing from a prolate to an oblate spheroid just before contact then both the impact and oscillation act together to flatten the drop.

### *Surface effects*

Good vortex rings are formed at dropping heights which approach zero (see, for example, figure 3*b*). These rings may obtain their energy from two sources: (1) surface energy of the drop or (2) potential energy of the drop. A simple calculation shows that the surface energy is roughly twice the potential energy for a drop placed on a liquid surface (assumed to fall freely through a distance equal to the drop radius). It seems reasonable then to assume that the surface energy contributes considerably to the vortex ring energy.

If the kinetic energy of the falling drop contributed significantly to the energy of the vortex ring, we should expect the rings produced from $H_2$ to be much more energetic than those produced from $H_1$. Reference to figures 1, 3*b* and 4 shows that this is in fact not the case. Thomson & Newall showed, for very small water drops, that the penetration decreased with increasing order over five optimum heights.

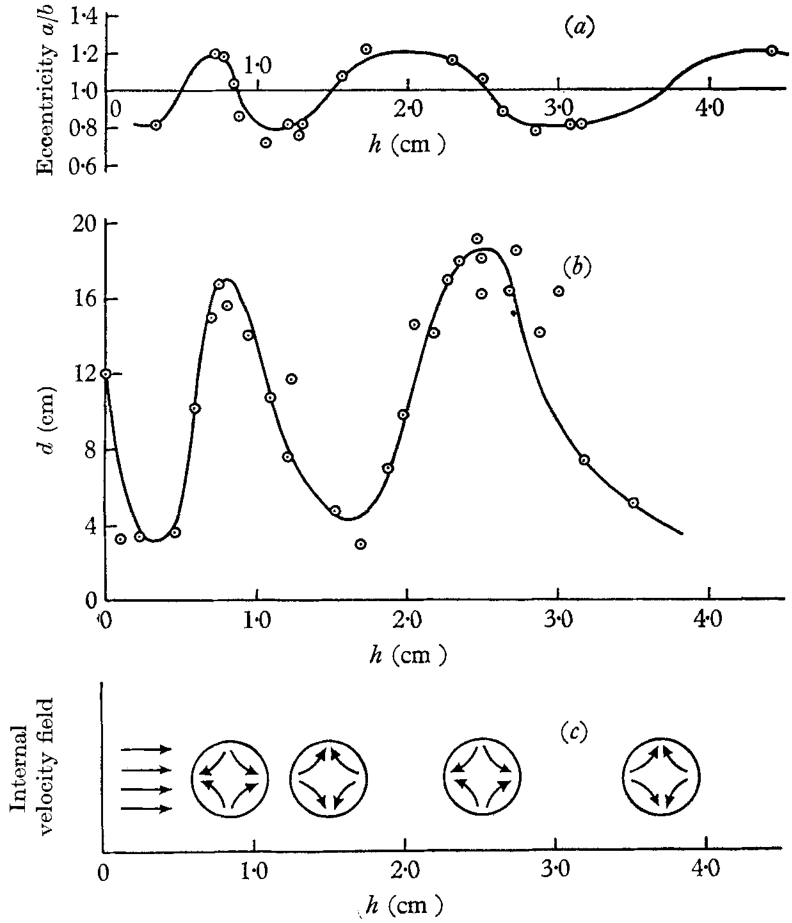


FIGURE 3. (*a*) Eccentricity $a/b$ of falling drop, (*b*) $d$–$h$ plot for the rings produced and (*c*) the internal velocity field in the spherical shape. Water drop of volume 0·06 $cm^3$ from conical tip.

Thus a drop, falling from not too great a height, on impact with a liquid surface is brought instantaneously to rest. The resulting vortex ring derives its energy mainly from the surface energy of the drop.

Unfortunately the core diameter of a vortex ring is not easily measurable so that no detailed calculations of its energy are possible.

### *Vortex ring velocity and the Reynolds number*

Two main reasons motivated the measurement of the vortex ring velocity: (1) to obtain a further comparison between the vortex rings produced from $H_1$ and $H_2$, and (2) to try to understand the anomalous results for the low-viscosity liquids, in terms of a Reynolds number.

The translational velocity of a water vortex ring is shown in figure 4 for a drop falling from zero height ($H_0$), $H_1$ and $H_2$. Vortex rings from $H_0$ are clearly inferior, possibly because of the drop's 'pear' shape on contact. The small difference in ring velocity for $H_1$ and $H_2$ is most likely a result of either differing departures from sphericity or small recoil velocities due to the different impact velocities. To a first approximation the rings from $H_1$ and $H_2$ form from very similar initial situations at the surface.

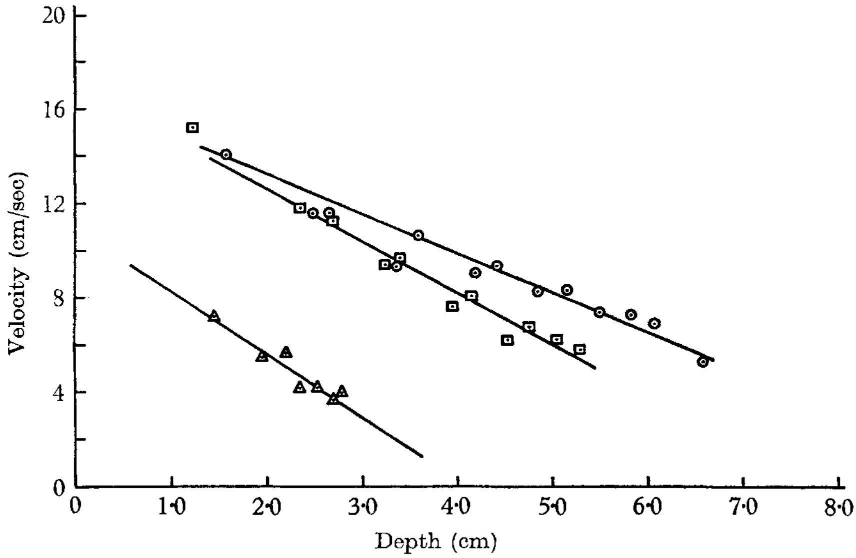


FIGURE 4. Translational velocity of vortex ring versus depth of penetration. Water: drop volume 0·06 cm³. △, $H_0$; ⊡, $H_1$; ⊙, $H_2$.

| Liquid | Viscosity (cm²/s) | Drop diameter (cm) | Reynolds number |
|---|---|---|---|
| Water | $1 \times 10^{-2}$ | 0·48 | 800 |
| Benzene | $7{\cdot}4 \times 10^{-3}$ | 0·36 | 700 |
| Acetone | $4{\cdot}1 \times 10^{-3}$ | 0·34 | 1100 |

TABLE 2. The Reynolds numbers $R$ for vortex rings

The most meaningful Reynolds number ($R$) for the destruction of a vortex ring is that obtained using the translational velocity, ring diameter and the kinematic viscosity of the fluid. As the ring penetrates the bath its velocity decreases and its diameter increases. The photographs show that the product (velocity) × (diameter) decreases with increasing penetration. The maximum Reynolds numbers are thus obtained using the initial velocity and diameter. The initial velocity is obtained from figure 4 by extrapolation to zero penetration. This is somewhat arbitrary since the ring presumably takes a finite distance to form. The initial diameter may also be obtained by extrapolation of the ring photographs. Intuitively one would expect the initial diameter to be closely equal to the drop diameter. The data confirm this. The Reynolds numbers for water, benzene and acetone are given in table 2, for rings of greatest penetration.

No definite conclusions can be formed regarding the 'double-peaks' observed in acetone and Freon 11. However, with a Reynolds number greater than 1100 it would not be surprising for a ring to become unstable, and thus disintegrate.

### *Model of ring formation*

Consider a liquid drop just detaching from a tip. As the drop falls it will oscillate about a spherical shape. If it strikes a liquid surface when it has a spherical shape and when it is changing from an oblate to a prolate spheroid, then the internal velocity field of the drop will tend to maintain the spherical shape. Probably the translational velocity of the drop is reduced to a small value.

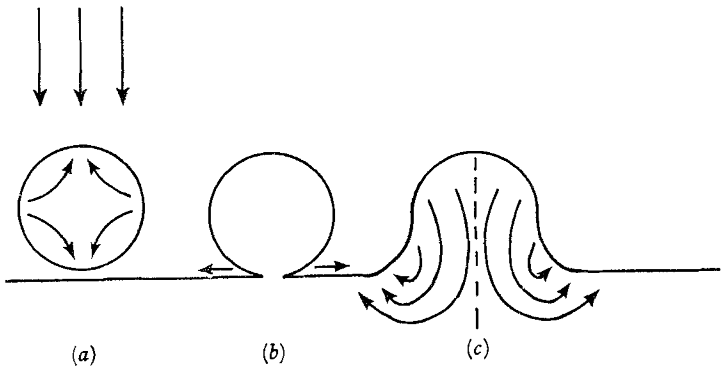


FIGURE 5. Liquid drop (*a*) before, (*b*) during and (*c*) after contact with a bath surface.

Figure 5 shows a spherical drop (*a*) before contact, (*b*) during contact and (*c*) after contact with the liquid surface. If surface tension is important then we can visualize the formation of the vortex ring as follows. In figure 5*c* the curvature of the free surface is such that the liquid comprising the drop is experiencing a relatively high pressure, whereas the liquid in the 'neck' region is at a relatively low pressure. The fluid elements of the drop are accelerated as shown, the pressure gradients coinciding with the normals at the free surface. Thus the necessary circulation is produced and a vortex ring results.

## Conclusions

The formation of vortex rings by a liquid drop falling into a bath has been investigated. Optimum dropping heights have been observed in agreement with previous experiments. Successive optimum heights are in general found to differ by time intervals equal to the drop oscillation time. A photographic study shows that there are two conditions for optimum vortex ring formation; the drop must be spherical and it must be changing from an oblate to a prolate spheroid upon contact with the bath.

Some anomalous results were seen with liquids of low viscosity. The Reynolds number for the vortex rings are approximately 1000 for the liquids investigated. In many instances the critical value for the Reynolds number is only slightly greater than 1000, so that it is not unreasonable to explain the anomalous results by assuming that the rings disintegrate immediately after formation.

A model has been proposed for the formation of a ring based on the pressure gradients resulting from the surface tension of the liquid. The previously proposed mechanism, by Thomson & Newall, is based on viscosity and vorticity diffusion. The experiments do not prove or disprove either model. Conclusive evidence may be forthcoming from high-speed photography of the drop/vortex ring transition.

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